USE OF MONTE CARLO SIMULATIONS IN VALUATION

NEIL BEATON and JOHN SAWYER
Alvarez & Marsal Valuation Services LLC

Introduction

Valuation professionals are constantly presented with challenging client needs, such as valuing new, and often exotic, securities with a variety of features that defy commonly-used valuation techniques. Monte Carlo simulations, however, can help bridge the gap between ordinary and extraordinary valuation assignments. The Monte Carlo analysis arose out of computer simulations created to address equilibrium properties for specific experiments. Prior to the advent of computers, the outcome of an experiment could be predicted in only one way: by making use of a theory that provided an approximate description of the system under consideration. An approximate theory was used because very few model systems could compute exact equilibrium properties. As a result, most properties of real materials were predicted on the basis of approximate theories. However, approximate theories required one to execute an experiment and then compare the results with the thesis. This was suboptimal because such experimentation was often expensive, and feedback took so much time to gather. With the advent of computer simulation, researchers were able to obtain very accurate results for a given model system without having to rely on approximate theories. It is from this original work that Monte Carlo has found its way into the world of valuation.

Pulling Back the Curtain

Often Monte Carlo is seen as a more sophisticated method for valuing an asset or liability with a level of complexity that traditional valuation approaches or methods seemingly are unable to capture. However, the first step in implementing Monte Carlo is to understand that it is NOT a distinct valuation approach or method and does not provide a solution to valuing any asset or liability unless the underlying economics are understood and input correctly. Rather than being a valuation approach or method, Monte Carlo is a technique for performing a set of calculations for the general purpose of understanding/measuring the impact of one or more, often uncertain, variables on the outcome of those calculations, which may represent either a final output or an input into further calculations.

In the implementation of a Monte Carlo simulation, certain distribution and/or correlation assumptions are applied to one or more variables of a calculation. Then, hundreds or thousands of trials are conducted in which a different combination of input variables is selected based on the distribution and/or correlation assumptions. The outcomes are recorded for each trial, enabling a statistical analysis of all the trials of the simulation.

For example, a company's potential future cash flow could be analyzed by applying certain distribution and correlation assumptions to the variables impacting its financial performance, such as product price, quantity sold, fixed and variable costs, etc. Taken a step further, an appraiser could introduce discounted cash flow calculations into the simulation to measure the uncertainty of the cash flows and/or derive a value of the subject company.

While a Monte Carlo simulation is an extremely powerful tool for measuring and obtaining insight into uncertainty, the above example can also be illustrative of the limitations of this technique.

First, the statistics (outputs) produced by the simulation are meaningless if the distributions and correlations of the variables (inputs) are not well supported. Garbage in, garbage out, as the proverb goes.

Second, the Monte Carlo simulation and the resulting statistics may provide a false sense of accuracy or ability to capture risk and potentially not be any more insightful than a simple data table or scenario-based analysis (both easily accomplished with tools available in a standard spreadsheet application).

Finally, the mean statistics (typically utilized for estimating the value of an asset or liability) produced for any given outcome/result may not be meaningfully different than would be produced by using static calculations based on the mean of the underlying inputs/variables. This is particularly true when the outcome/result varies linearly without any upper or lower bounds. Thus, a Monte Carlo simulation may not be as beneficial for certain calculations, particularly when considering the relatively complex and time-intensive nature of implementation.

The situations in which Monte Carlo is most useful – and often required – are when attempting to analyze/value an asset or liability with outcomes that are path-dependent, contingent, conditional, and/or non-linear (e.g., fixed outcomes conditional on a variable underlying metric, outcomes with minimums or maximums, etc.). A brief description of each condition follows.

Path-dependent outcomes are dependent on the measurement of certain results or performance over time. For example, a restricted stock award may vest only when the underlying stock price reaches a defined threshold during a defined period; thus, the stock prices through time and not just at maturity dictate the value of the award.

Contingent and/or conditional outcomes are dependent on the occurrence of certain circumstances or results. For example, an acquiring company may offer a fixed earn-out payment to a target company based on the future achievement of a minimum earnings target.

Non-linear outcomes are those in which the outcome is not proportional to the underlying asset/liability. For example, a typical stock option only provides a positive payoff if the underlying stock price exceeds the exercise price at maturity and results in zero value in all other scenarios – the payoff is non-linear with respect to the underlying stock price.

These conditions are most often encountered in the valuation of equity or debt derivatives (such as restricted stock, options, and warrants with anti-dilution provisions) and other complex financial instruments in which the outcomes or payoffs generally meet one or more of these criteria. Occasionally, a Monte Carlo simulation is employed even in the absence of these conditions, when implementation into a standard closed-form solution, such as a binomial lattice model, may be too complex and difficult with standard spreadsheet software.

In short, Monte Carlo should not be considered a magical solution to valuing an asset or liability. The variables determining the outcome or payoffs need to be understood; then it should be determined whether the outcome or payoff has any path-dependent, contingent, conditional, or non-linear outcomes that cannot be properly measured using closed-form or other mathematical solutions. Once these questions have been answered, it may then be appropriate to consider a Monte Carlo simulation to address the problem at hand.

Understanding Key Statistics and Conducting Diagnostics

After understanding when and how to apply Monte Carlo simulations for valuation purposes, it is important to be able to interpret the resulting statistics of the simulation and conduct diagnostics using those statistics to ensure the simulation is performing as expected.

During the preparation of the analysis/model to be used in the Monte Carlo simulation, the user should have some expectations of the performance of the simulation and results, and then identify and design diagnostics that will facilitate a statistical analysis of the results.

In understanding statistics for any Monte Carlo simulation, it should be reiterated that within the simulation, each trial is of equal weight (if a certain outcome is more probable than others, then that outcome will occur in more trials than others). Thus, the statistical analysis is performed on the entire dataset of the outcomes from all trials within the simulation with each outcome given equal weight.

The following is a description and summary of how to interpret some key statistics that may be relevant when performing a Monte Carlo simulation:

Mean – The mean of the results, in most cases, is the conclusion to derive the input into another calculation (i.e., discrete cash flow when simulating financial statements) or the estimate of value; therefore, the mean is the most critical statistic for valuation purposes (but not the only).

Median – In certain instances, the median may be considered a more meaningful indication of the "average" of a distribution than the mean, given that it is less skewed by outliers. In the context of a Monte Carlo simulation, the median can be helpful in understanding the distribution of the results. As an example, in a unimodal distribution if the mean is less than the median, this indicates that the mean is not in the middle of the distribution, but instead the distribution is skewed to the left. Additionally, certain accounting guidance, such as determining the average time to vesting for market-based stock awards, may require the use of the median for a particular outcome.

Minimum/Maximum – The minimum and maximum are helpful to understand the potential range of outcomes as well as to ensure the simulation is not producing illogical results (e.g., the value of a restricted stock

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award or option should never result in a negative value or a security with a fixed payoff should not have results exceeding the fixed amount).

Standard Deviation – The standard deviation is helpful to understanding the general distribution of the results; a larger standard deviation indicates a wider distribution of results. The expectation regarding the standard deviation of any outcome should be consistent with the underlying assumptions (e.g., higher expected volatility of stock price should correspond with a higher standard deviation of outcomes) and complexity of payoff structure, vesting, etc.

Kurtosis – The kurtosis is the measure of the extent the distribution of the results is peaked or flat. The notable value is 3.0, which indicates a standard normal distribution. A kurtosis higher than 3.0 indicates the results are peaked and concentrated at the mean and less than 3.0 indicates the results are relatively flat at the mean.

Skewness – The skewness statistic provides a numerical representation of what any observer of a distribution chart would be able to note. A skewness of 0 indicates a symmetrical distribution of results, while a positive value indicates a log-normal or skewed to the left distribution.

While it may be tempting to prepare the Monte Carlo simulation and just pull the mean from the results to derive the estimate of value without further analysis, it has been our experience that a more detailed review of the statistics and advanced consideration of potential diagnostics can provide assurances that the simulation is performing as expected and allow an analyst to provide insightful explanations of the results that may be invaluable when discussing with stakeholders.

An Example of Application

As is the case with most new concepts, an example is often helpful to fully understand and apply the concept - certainly Monte Carlo simulations are no different. Thus, we are using the valuation of a relative total shareholder return restricted stock award (commonly referred to as an "rTSR") to illustrate how to implement and interpret the results of a Monte Carlo simulation.

In our example, the rTSR award's vesting will be based on the subject company's stock price relative to a group of four peer companies. The vesting percentage is based on rank of return (calculated using the 20-trading day average prior to the grant date and preceding the maturity date) over the measurement period (two years), as follows:

Rank 1st – 200 percent of shares

Rank 2nd – 150 percent of shares

Rank 3rd – 100 percent of shares

Rank 4th – 50 percent of shares

Rank 5th – 0 percent of shares

The above vesting conditions contain both conditional (rank of return) and non-linear (shares vesting dependent on rank and the value of the award is not linear with stock price) outcomes; thus, as detailed previously, the valuation of the rTSR award requires a Monte Carlo simulation.

In order to value the rTSR award, simulating the stock price of the subject company and the four peer companies is required. The most common and widely accepted method for doing so is the geometric Brownian Motion (GBM). GBM utilizes a beginning stock price (S0), risk-free rate (μ), expected volatility of underlying stock (Đ), and simulated variable (Đ, a random number that has a normal distribution with a mean of zero and standard deviation of one) as inputs to the following formula to simulate each company's stock price:

The simulation can either be done using daily time-steps or, more efficiently, using a one-time jump to the beginning of the 20-day period at the maturity then using daily time-steps (t is the time interval of the time-step).

One additional element to consider is the correlation between the subject company's and each peer company's stock price as these types of awards often use companies within the same industry and some level of positive correlation would be expected. Based on our experience, correlation can have a meaningful impact on the results of an rTSR award and thus we incorporate the correlation in our analysis. The correlation of the simulated stock prices for each company are addressed by applying a correlation matrix to the simulated variable (ε) for each company for each time-step. This is typically calculated based on the historical correlation of daily stock price returns between the subject company and each of the peer companies, and between each peer company and all the other peer companies.

A Monte Carlo simulation consists of a large number (hundreds of thousands are typically necessary to capture the potential variability of the outcomes) of "trials" in which a new set of simulated variables (ε in our example) are selected based on defined distributions (a normal distribution is a frequently utilized distribution; however, there are many available distributions, such as log-normal, bi-modal, triangular, uniform, etc., that may be more appropriate for any specific simulated variable).

In our example, one trial would consist of a stock price path between the valuation date and the maturity (two years) for the subject company and each peer company, representing one potential outcome or scenario. In each trial, the return – based on the 20-trading day average preceding issuance and the simulated 20-trading day average preceding the maturity – and rank for each company would be calculated and used to determine the number of shares of the rTSR award vesting, and then the future value (shares vested multiplied by the future stock price) and present value (future value of rTSR award discounted at the risk-free rate) of the award would be determined.

At least one metric should be identified then tracked and recorded for each trial, enabling a review and interpretation of the results of the Monte Carlo simulation using statistical analysis. Often, we will track several metrics within the analysis to allow us to evaluate whether the simulation is performing as expected and further understand how various assumptions/factors might be impacting the results. For example, we might track the number of shares of the subject rTSR award vesting in each trial to ensure the minimum is not less than zero and maximum is not greater than 200 percent of the total award and understand the frequency/probability of reaching each vesting threshold (tracking the rank for each company would provide some insight here as well).

In addition to analyzing the results of a key outcome(s) to derive the intended value estimate, the statistical analysis can be leveraged for other outcomes within the simulation to interpret the performance of calculations and understand the results. Leveraging the example of the application of a Monte Carlo simulation for the valuation of an rTSR award, we can provide several examples of diagnostics that could be conducted for such an analysis.

One outcome of the rTSR simulation that would be of interest to analyze is the number of shares vesting and/or the rank of the subject company's stock price return. A simple solution would be to track the rank and/or number of shares vesting in each trial; however, the statistical analysis of the rank or number of shares vesting would not necessarily provide a clear understanding of the frequency of the various vesting thresholds (i.e., rank of return) being achieved. Alternatively, a secondary calculation could be performed which would result in a value of 1 when a certain rank is achieved and 0 if not; the resulting mean of all trials would provide the probability of that rank being achieved.

Another diagnostic that is often helpful to perform when preparing a valuation of an equity security or derivative using a risk-neutral framework (i.e., geometric Brownian Motion) is to calculate the present value (discounted at the risk-free rate) of the payoff of a standard European stock option (maximum of 0 and future stock price less exercise price) then compare the mean of the results to the value indicated by a standard Black-Scholes-Merton option pricing model with the same assumptions, which would provide some reassurances that the simulation of the stock price is behaving as expected and/or a sufficient number of trials has been selected. Alternatively, the behavior of the stock price simulation can be assessed by comparing the mean of the results of the present value of the future stock price at maturity in each trial to the beginning stock price; the theoretical difference should be zero.

In the subject example, the key metric to track would be the resulting present value of the rTSR award for each trial as the mean of all the trials would represent the conclusion of the fair value or fair market value of the rTSR award (each trial is equally likely and, therefore, given equal weight). Additionally, it might be necessary to track the time to vesting for awards with variable maturities to capture the median term for certain accounting disclosures under financial reporting.

In conclusion, Monte Carlo simulations can be useful and powerful tools for the valuation analyst tackling complex problems that don't lend themselves to commonly-used valuation techniques. Once the Monte Carlo framework is understood, relevant inputs can be identified and simulated to provide statistically valid results that can enhance most valuation assignments.

ABOUT THE AUTHORS

Neil Beaton

Neil is a Managing Director with Alvarez & Marsal Valuation Services, LLC. Previously, he was the Global Lead of Complex Valuation with Grant Thornton LLP. He has over 25 years of experience analyzing both closely and publicly held companies. Neil has appeared as an expert witness across the country, is a frequent lecturer at local universities, an instructor for the AICPA's business valuation courses, and speaks nationally on business valuation with a special emphasis on early stage and high technology companies. He has served on a number of AICPA Committees and Task Forces. Neil has a Bachelor of Arts Degree in Economics from Stanford University and a Master of Business Administration in Finance from National University. In addition to his formal education, Neil is a Certified Public Accountant, Accredited in Business Valuation, Certified in Financial Forensics, a Chartered Financial Analyst and an Accredited Senior Appraiser in business valuation from the American Society of Appraisers.

John Sawyer

John Sawyer is a Senior Director with Alvarez & Marsal Valuation Services, LLC. Prior to joining A&M, John worked in Grant Thornton LLP's valuation service practice and Ernst & Young's complex securities group, where he focused on quantitative analysis of financial instruments and early stage company valuation. He specializes in the valuation of public and closely-held business and business segments, complex financial instruments, and related matters for financial statement reporting, tax reporting, corporate planning, litigation support, and other purposes. John has been involved with research and valuing companies in a variety of industries, including energy, green-tech, aerospace, blockchain, battery-technology, electronic vehicles, biotechnology and pharmaceutical, cryptocurrency mining, Internet, software, and a variety of other technology segments. John earned a Bachelor of Arts Degree in Business Administration, with a Finance Concentration, from Washington State University.

AIRA | Association of Insolvency & Restructuring Advisors

221 W. Stewart Avenue, Suite 207
Medford, OR 97501
Phone: 541-858-1665
Fax: 541-858-9187
aira@aira.org
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